TURLI CHEGARALANISHLI HOL UCHUN Π STRATEGIYANING QURILISHI HAQIDA

Abduraximova Zulayxo Ikromjon qizi
Turan xalqaro universiteti o'qituvchisi
zulayxoabduraximova96@gmail.com

Annotasiya: *Ushbu maqolada Π strategiyaning geometrik chegaralanish, integro – geometrik chegaralanishli holatlar uchun qurilishi yoritilgan. Π strategiyaning qurilishi turli chegaralanishli hol uchun quvish – qochish maslasining yechimlariga uzviy bog'liqligi isbotlangan.*

Kalit so'zlar: *Π strategiya, geometrik chegaralanish, integro – geometrik chegaralanish.*

О ПОСТРОЕНИИ Π – СТАТЕГИИИ ДЛЯ РАЗЛИЧНЫХ ГРАНИЧНЫХ СЛУЧАЕВ

Аннотация: *В статье описано построение Π стратегии для геометрически ограниченных, интегро-геометрически ограниченных случаев. Доказано, что построение Π-стратегии неразрывно связано с решением задачи погони-убегания для различных предельных случаев.*

Ключевые слава: *Π стратегии, геометрическое ограничение, интегро - геометрическое ограничение*

ON THE CONSTRUCTION OF Π STRATEGY FOR DIFFERENT BOUNDARY CASES

Annotation: *This article describes the construction of strategy P for geometric bounded, integro-geometrically bounded cases. It has been proved that the construction of P strategy is inextricably linked to the solutions of the chase-escape problem for different limiting cases.*

Keywords: *Π strategy, geometric limitation, integro - geometric limitation.*

Differensial o’yinlar nazariyasida chegaralanishlar uchun Π strategiya muhim ahamiyatga ega. Har bir chegaralanish uchun alohida Π strategiyalar mavjud. Quyida ularni har birini ko’rib chiqamiz.

Fazoda P va E obyekt harakatlanyapti

P: $\begin{cases} \dot{x} = U \\ x(0) = x_0 \end{cases}$

(1)

E : $\begin{cases} \dot{y} = V \\ y(0) = y_0 \end{cases}$ (2)

1. Geometrik chegaralanish :

$a)\ |U| \le \alpha\ ;\ \ |V| \le \beta\ ;\ \ |U| = \sqrt{U_1^2 + \cdots + U_n^2}\ ; |V| = \sqrt{V_1^2 + \cdots V_n^2}$

$b)\ U = (U_1; U_2) \to |U_1| \le \alpha_1\ , |U_2| \le \alpha_2$

$V = (V_1; V_2) \to |V_1| \le \beta_1\ , |V_2| \le \beta_2$

Π strategiya : $U(v) = v - \lambda_G(v)\xi_0$, $\xi_0 = \frac{x_0 - y_0}{|x_0 - y_0|}$ bu yerda

$$\lambda_G(v) = < v, \xi_0 > + \sqrt{< v, \xi_0 >^2 + \alpha^2 - |v|^2}$$

$\alpha > \beta$ da tutish masalasi yechiladi ; $\alpha \le \beta$ da qochish masalasi yechiladi .

2. Integro – geometrik chegaralanishli xol :

a) $\int_0^\infty |U(s)|^2 ds \le \rho_0,$ (3)

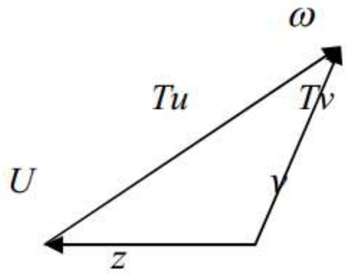
$|v| \le \beta$ (4)

b) $\int_0^t |U_1(s)|^2 ds \le \rho_{10},\ \int_0^t |U_2(s)|^2 ds \le \rho_{20}$

$|v_1| \le \beta_1,$ $\qquad |v_2| \le \beta_2$

P: $\dot{x} = U(t)$, $x(0) = x_0 \to x(t) = x_0 + \int_0^t U(s)ds$

E : $\dot{y} = V(t)$, $y(0) = y_0 \to y(t) = y_0 + \int_0^t V(s)ds$



Tutish masalasi : $\forall t^* : x\left(t^*\right) = y\left(t^*\right)$ (5)

Qochish masalasi : $x(t) \neq y(t)\ ,\ t \geq 0$ (6)

$$z + Tu = Tv\ ,\ Tu = Tv - z\ ,\ u = v - \frac{z}{T} \quad (7)$$

$$\int_0^T |U|^2 ds = \rho, T|U|^2 = \rho, |U|^2 = \frac{\rho}{T} \quad (8)$$

Quyidagicha belgilash kiritib olamiz : $\frac{|z|}{T} = \lambda \Rightarrow U = v - \lambda\xi, |U|^2 = \lambda\frac{\rho}{|z|}$

$$\begin{cases} |U|^2 = |v|^2 - 2\lambda\langle v, \xi\rangle + \lambda^2 \\ \lambda\frac{\rho}{|z|} = |v|^2 - 2\lambda\langle v, \xi\rangle + \lambda^2 \end{cases} \Rightarrow \lambda^2 - 2\lambda\left(\frac{\rho}{2|z|} + \langle v, \xi\rangle\right) + v^2 = 0$$

(9)

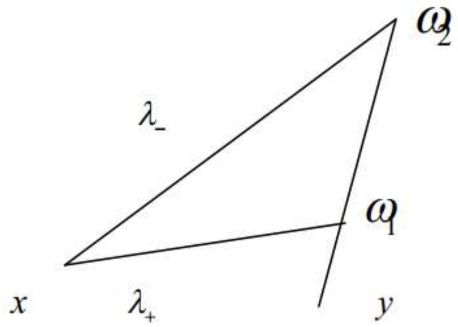
$$\lambda_{1.2} = \frac{\rho}{2|z|} + \langle v, \xi\rangle \pm \sqrt{\left(\frac{\rho}{2|z|} + \langle v, \xi\rangle\right)^2 - |v|^2}$$

(10)

λ_1 va λ_2 ham musbat . Savol tuo'iladi qaysi λ ni olamiz ?

$\lambda_1, \lambda_2 > 0$ agar , $\frac{\rho}{2|z|} + \langle v, \xi\rangle \succ 0$ bo'lsa yechim mavjud bo'lishi uchun ildiz osti

aniqlangan bo'lishi kerak ya'ni $D = \left\{(\rho, z, v) : \left(\frac{\rho}{2|z|} + \langle v, \xi\rangle\right)^2 - |v|^2 \geq 0\right\}$



Integro – geometrik holda quvlovchini parallel yaqinlashtirish orqali (ω_1, ω_2) oraliqda tutish mumkin bo'lar ekan . Biz bitta quvlovchi bitta qochuvchi bo'lgan holda ko'ramiz ya'ni λ_+ ni olamiz . Shuning uchun (10) ga yechim sifatida λ_1 ni tanlaymiz . Natijada hal qiluvchi funksiya deb,

$$\lambda(\rho, z, v) = \frac{\rho}{2|z|} + \langle v, \xi \rangle + \sqrt{(\frac{\rho}{2|z|} + \langle v, \xi \rangle)^2 - |v|^2}$$

(11)

funksiyani olamiz . n o'lchovli fazoda $2n+1$ ta yechimi bor. (11)- funksiyani aniqlanish sohasini tahlil qilamiz $D = \left\{ (\rho, z, v) : (\frac{\rho}{2|z|} + \langle v, \xi \rangle)^2 - |v|^2 \right\}$

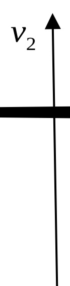
$(\frac{\rho}{2|z|} + \langle v, \xi \rangle + |v|)(\frac{\rho}{2|z|} + \langle v, \xi \rangle - |v|) \geq 0$ demak ,

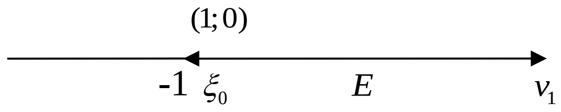
$$\frac{\rho}{2|z|} + \langle v, \xi \rangle - |v| \geq 0 \qquad (12)$$

bo'lishi kerak . (12) tengsizlikning geometrik mohiyati (v ga nisbatan):

$a^2 = \frac{\rho}{2|z|}$, $a^2 + v_1\xi_1 + ... + v_n\xi_n - \sqrt{v_1^2 + ... + v_n^2} \geq 0$ figura hosil bo'ladi .

Soddalashtirish uchun tekislikda ko'raylik . $a^2 + v_1\xi_1 + v_2\xi_2 - \sqrt{v_1^2 + v_2^2} \geq 0$



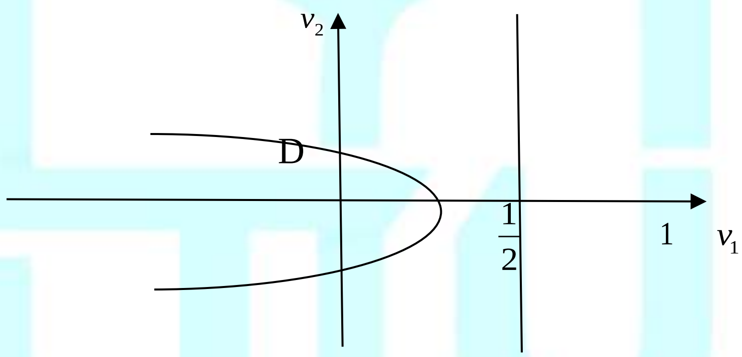


$\xi_0 = \frac{x_0 - y_0}{|x_0 - y_0|}, x_0 = (0;0), y_0 = (1;0); \xi_0(-1;0), \xi_1 = -1, \xi_2 = 0$

$a^2 - v_1 - \sqrt{v_1^2 + v_2^2} \geq 0$, $a^2 \geq v_1 + \sqrt{v_1^2 + v_2^2} \geq 0$, $a = 1$ desak , $v_1 + \sqrt{v_1^2 + v_2^2} \leq 1$,

$\sqrt{v_1^2 + v_2^2} \leq 1 - v_1$

$1 - 2v_1 + v_1^2 \geq v_1^2 + v_2^2, 1 - 2v_1 \geq v_2^2, v_1 \leq \frac{1 - v_2^2}{2}$



D sohamiz parabolaning ichi ekan . Uch o'lchovli fazoda ko'rsak , paraboloid bo'lib qolar ekan . Agar $a \neq 1$ bo'lsa uchi $\frac{a}{2}$ da bo'lar ekan tekislikda .

$\xi_0(-1,0,0)$; $a^2 - v_1 - \sqrt{v_1^2 + v_2^2 + v_3^2} \geq 0, a^2 - v_1 \geq \sqrt{v_1^2 + v_2^2 + v_3^2}$

$a^4 - 2a^2 v_1 + v_1^2 \geq v_1^2 + v_2^2 + v_3^2, a^4 - 2a^2 v_1 \geq v_2^2 + v_3^2, v_1 \leq \frac{a^4}{2a^2} - \frac{v_2^2 + v_3^2}{2a^2}, v_1 \leq \frac{a^2}{2} - \frac{v_2^2 + v_3^2}{2a^2}$

$\beta \leq \frac{a^2}{2} = \frac{\rho}{4|z|} \Rightarrow \rho \geq 4\beta|z|$. Demak , xulosa (11) $\lambda(\rho, z, v)$ funksiya $\rho \geq 4\beta|z|$ bo'lganda $\forall |v| \leq \beta$ uchun aniqlangan .

(11) ko'rinishni yuqoridagi $U = v - \lambda\xi, |U|^2 = \lambda \frac{\rho}{|z|}$ tenglikka olib kelib qo'yamiz va quyidagi strategiyani hosil qilamiz :

$$\begin{cases} U(\rho, z, v) = v - \lambda(\rho, z, v)\xi \\ |U(\rho, z, v)|^2 = \lambda(\rho, z, v)\dfrac{\rho}{|z|} \end{cases}$$

(13)

(1) va (2) tenglamadan quyidagi tenglamani hosil qilamiz

$$\begin{cases} \dot{z} = U - v \\ z(0) = z_0 \end{cases} \quad (14)$$

bu yerda $z = x - y, \dot{x} = U; z_0 = x_0 - y_0, \dot{y} = v; \dot{x} - \dot{y} = U - v$

(14) ko'rinishdagi U ni o'rniga (13) ni 1-qatorini qo'yamiz natijada :

$$\begin{cases} \dot{z} = -\lambda(\rho, z, v)\xi \\ z(0) = z_0 \end{cases} \quad (15)$$

$$\rho(t) = \rho_0 - \int_0^t |U(s)|^2 ds$$

(16)

(16) tenglamaga $|U|^2$ ni o'rniga (13) ni 2-qatorini qo'yamiz

$\rho(\mathrm{t}) = \rho_0 - \int_0^t \lambda(\rho(s), z(s), v(s))\dfrac{\rho(s)}{|z(s)|} ds$ tenglikni hosil qilamiz . Endi har ikki tomondan hosila olaylik

$$\begin{cases} \dot{\rho}(t) = -\lambda(\rho, z, v)\dfrac{\rho}{|z|} \\ \rho(0) = \rho_0 \end{cases} \quad (17)$$

(15) va (17) lardan quyidagi natijaga ega bo'lamiz

$$\begin{cases} \dot{z} = -\lambda(\rho(t), z(t), v(t))\dfrac{z(t)}{|z(t)|} \\ \dot{\rho} = -\lambda(\rho(t), z(t), v(t))\dfrac{\rho(t)}{|z(t)|} \\ z(0) = z_0, \rho(0) = \rho_0 \end{cases}$$

(18)

(18) dan (ρ, z) ga nisbatan differensial tenglamalar sistemasi hosil bo‘lishi kelib chiqdi.

(18) sistema nochiziqli differensial sistema . Agar $z \neq 0$ bo‘lsa bu sistemaning o‘ng tomonidagi funksiya $\rho \geq 4\beta|z|$ shartda uzluksiz ((ρ, z) ga nisbatan) . Funksiya *t* ga nisbatan esa o‘lchovli funksiya (ya’ni Karatedore shartlari bajarilyapti) deyiladi. Karatedore sharti bo‘lishi uchun , Karatedore tenglamasining o‘ng tomoni *z* bo‘yicha Lipshist shartini qanoatlantirishi kerak ya’ni

$$|f(z_1, t) - f(z_2, t)| \leq L|z_1 - z_2| \quad (19)$$

(18) tenglamaning o‘ng tomonidagi funksiyalar (ρ, z) bo‘yicha $z \neq 0$ holda Karatedore shartlari o‘rinli . Shuning uchun (18) sistemani yagona yechimi mavjud bo‘ladi . (18) tenglamadan quyidagicha almashtirish hosil qilamiz

$$\begin{cases} \dot{z}_1 = -\lambda(\rho(t), z(t), v(t))\dfrac{z_1(t)}{|z(t)|} \\ \dot{z}_2 = -\lambda(\rho(t), z(t), v(t))\dfrac{z_2(t)}{|z(t)|} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dot{z}_n = -\lambda(\rho(t), z(t), v(t))\dfrac{z_n(t)}{|z(t)|} \end{cases} \Rightarrow \begin{cases} \dfrac{\dot{z}_1}{z_1} = -\lambda(\rho(t), z(t), v(t))\dfrac{1}{|z(t)|} \\ \dfrac{\dot{z}_2}{z_2} = -\lambda(\rho(t), z(t), v(t))\dfrac{1}{|z(t)|} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{\dot{z}_n}{z_n} = -\lambda(\rho(t), z(t), v(t))\dfrac{1}{|z(t)|} \end{cases}$$

endi integrallasak ,

$$\ln z_i \Big|_0^t = -\int_0^t \lambda(\rho(t), z(t), v(t)) \frac{1}{|z(t)|} ds; \; z_i(t) = z_i(0) e^{-\int_0^t \lambda(\rho(t), z(t), v(t)) \frac{1}{|z(t)|} ds}, \; i = \overline{1, n}$$

$e^{-\int_0^t \lambda(\rho(t), z(t), v(t)) \frac{1}{|z(t)|} ds} = h(t)$ deb olsak ,

$$z(t) = z_0 h(t) \tag{20}$$

hosil bo'ladi . Xuddi shu kabi ,

$$\begin{cases} \rho(t) = \rho_0 h(t) \\ z(t) = z_0 h(t) \end{cases} \tag{21}$$

bu yerda $h(t) = h(z, \rho, v, t)$. (21) ni (11) ga olib borib qo'yaylik

$$\lambda(\rho, z, v) = \frac{\rho}{2|z|} + \langle v, \xi \rangle + \sqrt{(\frac{\rho}{2|z|} + \langle v, \xi \rangle)^2 - |v|^2} = \frac{\rho_0 h}{2|z_0| h} + \left\langle v, \frac{z_0 h}{|z_0| h} \right\rangle + \sqrt{(\frac{\rho}{2|z|} + \langle v, \xi \rangle)^2 - |v|^2} = \lambda(\rho_0, z_0, v)$$

Demak , boshlano'ich holatdagi (ρ_0, z_0) ni bilsak bo'ldi ekan .

Ta'rif : Integro –geometrik differensial o'yinda Π strategiya deb , quyidagi funksiyaga aytamiz

$$U(v) = v - \lambda_{IG}(v) \xi_0 \tag{22}$$

bu yerda $\lambda_{IG}(v) = \frac{\rho_0}{2|z_0|} + \langle v, \xi_0 \rangle + \sqrt{(\frac{\rho_0}{2|z_0|} + \langle v, \xi_0 \rangle)^2 - |v|^2}$

Mavzu bo'yicha tarixiy ma'lumot : Strategiyalar quvish masalasi qadimdan olimlarni qiziqtirgan . Eramizdan 2000 yil avval Xitoy qo'lyozmalarida "Burgut va O'lja masalasi " o'rganilgan bunda burgut o'z o'ljasini izma –iz quvish strategiyasi orqali harakatlanib ushlash masalasi ko'rilgan . Lekin matematika rivoji u davrda bu masalani yechishga yetarli bo'lmagan . 1732- yil fransus gidrogrifi va matematigi

Bagauer " Tulki va Quyon " masalasini yechadi . Bunda quyin ma'lim to'o'ri chiziq bo'yicha qochib borganda tulki uni izma – iz quvish natijasida qaysi trayektoriya orqali harakat qilishi mumkinligi m α asaasi yechildi . Bu masala amerikalik olim Lokning " Zambaraklarni boshqarish " kitobida bafurcha tahlil qilinib yechimlari keltirilgan .

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